Symbolab Limits Cheat Sheet

Limit Properties:

If the limit of $f(x)$, and $g(x)$ exists, then the following apply:

- $\lim_{x\to a} x = a$
- $\lim_{x\to a}\left(f(x)\right)^c = \left(\lim_{x\to a} f(x)\right)^c$
- $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
- $\lim_{x\to a}[c \cdot f(x)] = c \cdot \lim_{x\to a}[f(x)]$
- $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
- $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, where $\lim_{x\to a} g(x) \neq 0$

Limit to Infinity Properties:

For $\lim_{x\to c} f(x) = \infty, \lim_{x\to c} g(x) = L$, the following applies:

- $\lim_{x\to c}[f(x) \pm g(x)] = \infty$
- $\lim_{x\to c}[f(x) \cdot g(x)] = \infty, L > 0$
- $\lim_{x\to c}[f(x) \cdot g(x)] = -\infty, \ L < 0$
- $\lim_{x\to c}\frac{g(x)}{f(x)} = 0$
- $\lim_{x\to \infty} ax^n = \infty, \quad 0 < a$
- $\lim_{x\to -\infty} ax^n = \infty, n$ is even, $a > 0$
- $\lim_{x\to -\infty} ax^n = -\infty, \quad n$ is odd, $a > 0$
- $\lim_{x\to \infty}\frac{c}{x^a} = 0$

Indeterminate Forms:

- 0^0
- ∞^0
- 1^∞
- $\frac{\infty}{\infty}$
- $\frac{0}{0}$
- $0 \cdot \infty$
- $\infty - \infty$

Common Limits:

- $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k$
- $\lim_{x\to\infty}\left(\frac{x}{x+k}\right)^x = e^{-k}$
- $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

Limit Rules:

- Limit of a Constant: $\lim_{x\to a} c = c$
- Basic Limit: $\lim_{x\to a} x = a$
- Squeeze Theorem: Let f, g and h be functions such that for all $x \in [a, b]$ (except possible at the limit point c), $f(x) \le h(x) \le g(x)$. Also suppse that $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = L$, then for any $c, a \le c \le b$, $\lim_{x\to c} h(x) = L$
- L'Hopital's Rule : For $\lim_{x\to a}\frac{f(x)}{g(x)}$, if $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$
- Divergence Criterion: If there exists two sequences, $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ with: $x_n \ne c$, $y_n \ne c$ and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c$, $\lim_{n\to\infty} f(x_n) \ne \lim_{n\to\infty} f(y_n)$, then $\lim_{x\to c} f(x)$ does not exist
- Limit Chain Rule: If $\lim_{u\to b} f(u) = L$, and $\lim_{x\to a} g(x) = b$, and $f(x)$ is continuous at $x = b$, Then: $\lim_{x\to a} f\big(g(x)\big) = L$